Depicting the quantum state of a trapped Bose-Einstein condensate

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The experimental realisation of Bose-Einstein condensation in dilute alkali gases has allowed us the possibility of studying the dynamics of quantum fields with great detail [1, 2]. In this context, experimental results at low temperatures have confirmed the robustness of the Gross-Pitaevskii equation (GPE) [3, 4] to describe a Bose-Einstein condensate (BEC). In this work we make use of the mean-field approximation to characterise the quantum state of a trapped condensate, and propose a way of visualising it in a phase-space representation.

With in the standard mean-field theory, the normalised Gross-Pitaevskii equation, has the following form

$$\mathrm{i}\frac{\partial\phi}{\partial t} = \left[-\nabla^2_{\mathbf{r}} + V_{trap}(x) + \frac{nU_0}{\hbar\omega_{trap}}|\phi|^2\right]\phi \tag{1}$$

where, n is the number of particles in the sample, $U_0 = 4\pi\hbar^2 a/m$; m is the mass of a single particle and a the scattering length and ω_{trap} is the frequency of the trapping potential. It is a well-known fact, that the GPE has the form of a nonlinear Schrödinger equation, where the nonlinearity arises from two-body collisions.

Considering equation (1) in the Schrödinger picture, it can be written as $\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\Phi\rangle = \hat{H}|\Phi\rangle$. It is possible to find the solution of such an equation by expanding the state vector $|\Phi\rangle$ in a Fock space

$$|\Phi\rangle = \sum_n A_n|n;t\rangle, \tag{2}$$

where the number state

$$|n;t\rangle = \frac{1}{\sqrt{n!}}\int \mathrm{d}x_1...\mathrm{d}x_n\Phi_n(x_1,...,x_n,t)\phi^\dagger(x_1)...\phi^\dagger(x_n)|0\rangle, \tag{3}$$

is a superposition of states produced by creating particles in the positions $x_1, ...x_n$ with a weighting function Φ_n. This state vector can be expressed in the Hartree approximation as

$$|\Phi\rangle = \sum_n \frac{\alpha_0^n}{n!}\exp\frac{-|\alpha_0|^2}{2}\left(\int \mathrm{d}x\Psi_n(x,t)\phi^\dagger(x)\right)^n|0\rangle. \tag{4}$$

The state given by equation (4) can be easily visualised using a phase-space representation. The quasiprobability function $Q(\alpha_r, \alpha_i) = |\langle\alpha, \{\Psi(x,t)\}|\Phi\rangle|^2$ provides us with a very intuitive way of picturing the features of the quantum state. It can be considered to describe probability debsities ans has the further advantage of being readily measurable by quantum tomographic techniques. In order to use this description, we need to define a reference coherent state using the n-particle eigenstate for the field with envelope $\Psi_{\bar{n}}(x,t)$, where $\bar{n} = |\alpha_0|^2$ has the meaning of the average particle number.

It is shown that the ground state of the condensate has a Q-function that corresponds to that of a squeezed state (Figure 1). We also analyse the evolution of the quantum state of a condensate undergoing a ballistic expansion. This results of great importance as several output coupling schemes for atom lasers or some measurement are done by letting the atomic clud expand in this way [1, 5]. In this case, the simulations show that the evolution leads to the generation of Schrödinger cat states [6].

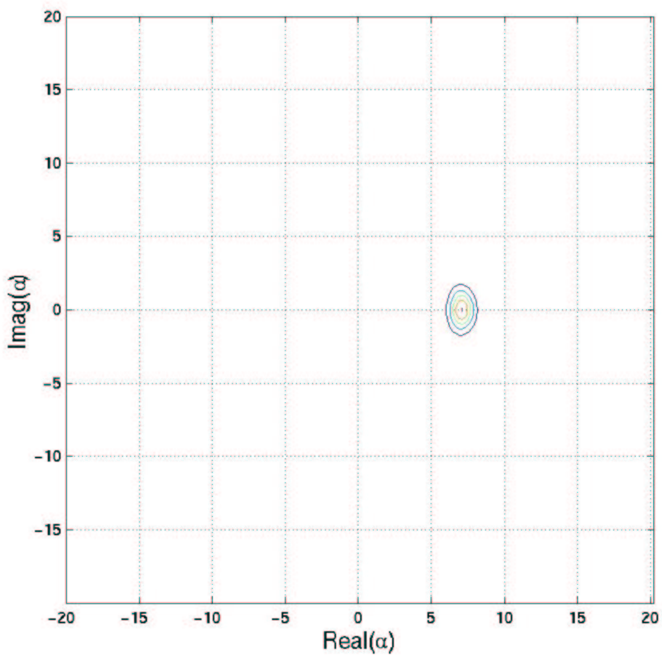

Figure 1: Q-function for Bose condensed gas described by the Thomas-Fermi approximation.

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