Investigating Quantum Chaos with Laser-Cooled Caesium Atoms

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Although there have been many investigations into classically chaotic systems, chaos in a quantum system is still a relatively ill-defined concept. We are, therefore, investigating the quantum analogue of a classically chaotic system in order to establish which aspects of classical chaos do persist in the quantum regime. This will clarify the features of 'quantum chaos' and thus define its meaning more precisely.

Classical chaos is characterised by the very sensitive dependence of the dynamics on the initial conditions. In a chaotic region of phase space, initially neighbouring trajectories diverge exponentially in time. In the quantum regime, however, it is not possible to use this definition because the unitary nature of any interaction necessarily implies that the overlap integral between two different wave functions is constant in time. This means that small changes in the initial state cannot yield divergent trajectories.

To overcome this apparent breakdown of the correspondence principle, a definition of quantum chaos in terms of the sensitivity of the system's evolution, for a given initial state, to small changes in the Hamiltonian has been proposed [1]. This is the approach that we have adopted. We consider the overlap of two states that are exposed to slightly different external potentials and examine it as a function of time. If the overlap decreases exponentially then the region of phase space occupied by the initial state can be said to be chaotic in the quantum sense.

One of the most extensively investigated systems in the field of classical chaos theory is the so-called *δ-kicked rotor*. Though conceptually simple it exhibits many important types of behaviour that are characteristic of chaos. A particle is subjected to periodic δ-function kicks from a potential, and the impulse received by the particle from a kick depends sinusoidally on its position at the instant of that kick. The effect of these kicks on the particle is similar to that of pushing a child on a swing. For the correct interval between successive pushes the child will receive the same impulse from each one.

We are investigating an experimental system that constitutes the quantum mechanical version of the δ-kicked rotor. In our system the particles (caesium atoms) have an initial momentum distribution that is Gaussian. The behaviour of the quantum δ-kicked rotor is quite different from its classical counterpart. Whereas the classical system exhibits diffusion of its momentum as kicks are applied, the quantum system displays the suppression of momentum diffusion known as dynamical localisation, and the momentum distribution becomes exponential in form. For special intervals between successive kicks, so-called 'quantum resonances' are observed. In such circumstances the system undergoes a significantly higher level of momentum diffusion than for non-resonant kicking intervals, and the momentum distribution is neither Gaussian nor exponential. Both types of behaviour can be accounted for in terms of the phase accumulated over the interval between consecutive pulses by the different momentum states populated by diffraction from the potential.

The *δ-kicked accelerator* is a modified version of the δ-kicked rotor in which an external symmetry-breaking force, in our case gravity, is present. Classically there is little difference between the behaviour of the two systems. Quantum mechanically, however, the behaviour of the accelerator is markedly different from that of the rotor. The effect of gravity in modifying the inter-kick phase accumulated by the momentum states populated through diffraction means that approximately 20% of the atoms receive a fixed momentum impulse from each standing wave pulse. This phenomenon is known as a 'quantum accelerator mode', and the momentum transfer to the system is much enhanced above that in the classical case. These modes occur for particular values of the kicking interval, close to those that yield quantum resonances in the rotor. The atoms in an accelerator mode do not form a continuous distribution but a momentum comb. Their de Broglie waves are also periodically localised in position space and so the accelerator mode isolates a region of phase space. This could, therefore, be used as a preparation technique to allow the characterisation of quantum phase space stability.

The quantum δ-kicked rotor and accelerator are realised in atom optics by the exposure of laser-cooled atoms to pulses of a standing wave of off-resonant, high-intensity light. These apply δ-function-like kicks to the atoms. Our experimental setup consists of a vacuum cell containing caesium atoms, which are trapped and cooled in a magneto-optic trap (MOT). The cloud of about 10^6 atoms, at a temperature of around 5μK, is released from the trap and the standing wave pulses are applied to it. Following this interaction the atoms fall freely through a sheet of light and their momentum distribution is measured by a time-of-flight technique.

Our current project is to develop the use of the accelerator mode as a means of isolating a specific region in phase space and characterising its stability. Two sets of pulses are applied to the atoms: the first set, in which gravity's effect is present, creates an accelerator mode and therefore yields a periodically localised atomic distribution in both position and momentum space. A $\pi/2$ burst of microwaves is then used to place these atoms in a superposition of two hyperfine levels, each of which will experience a different kick strength from further standing wave pulses. When the second pulse set, for which gravity's effect is absent, is applied each component of the superposition evolves under a different δ-kicked rotor Hamiltonian. Measurement of the population in one of these hyperfine levels after another $\pi/2$ microwave burst, with a variable phase relative to the first, allows the overlap (initially unity) between the wavefunctions of the two hyperfine levels to be ascertained. Measurement of the variation of this overlap as the number of standing wave pulses in the second set is varied will characterise the sensitivity of the system's evolution to variations in the Hamiltonian and hence the stability of the initially occupied region of phase space. This will be a clear manifestation of quantum chaos.

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