Scattering on a Bose–Einstein condensate

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Quantum mechanics textbooks usually include as one of the very first calculations a treatment of 1D scattering of a particle on a potential barrier. In this work we consider the mean-field analogy of this problem where the barrier consists of a Bose-condensate. When the particles of the condensate are identical to the incoming particle, proper care must be taken of the symmetry and the Bogoliubov treatment is needed. Depending on the momentum of the incoming particle, the effective barrier height varies and perfect transmission can occur at energies where a particle not identical with the condensate particles would need to tunnel to cross the mean field barrier.

Starting from a full description of the scattering we examine the applicability of the Bogoliubov approximation. Within this approximation, the problem takes on a form much like the usual potential barrier scattering but apparently with a number of new features due to the presence of both the u and the v functions. We describe various numerical methods for finding e.g. phase shifts as a function of the momentum of the incoming particle.

In many cases, it turns out that the main difference is contained, however, in the modified dispersion relation. Using semi-classical solutions with correct local wavenumbers gives in general qualitatively good approximations to the real wave functions. To calculate exactly the phase shifts and thus the transmission and reflection, one must go beyond the semi-classical solutions and we find simple and useful equations to describe the needed corrections. In Fig. 1 we show typical plots of e.g. the effective potentials corresponding to the correct local wavenumbers.

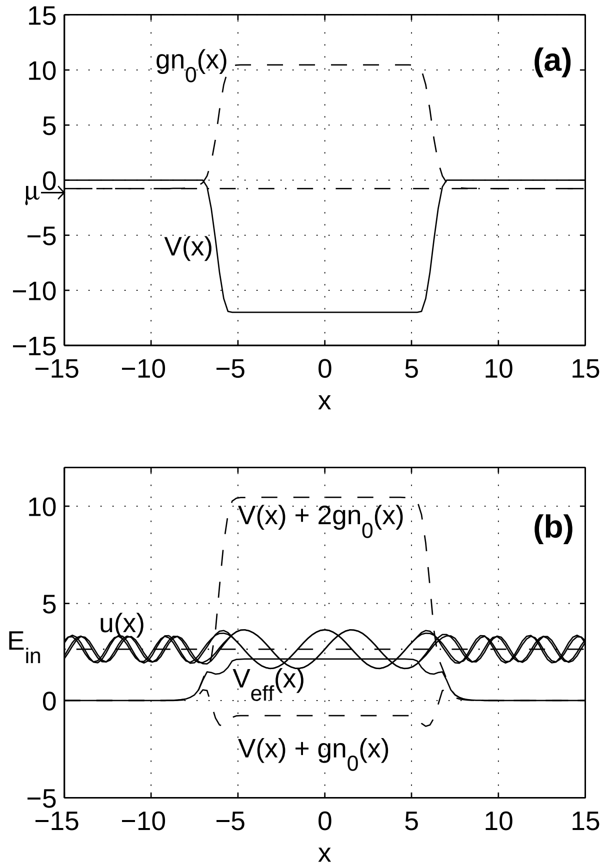

Figure 1: **(a)**: External potential and the condensate density, $n_0(x)$, for 1.4×10^4 atoms at a scaled interaction strength of $g = 0.01$. The condensate can only barely be held in the potential ($\mu \sim -0.8$). **(b)**: Effective potential, $V_{eff}(x)$, seen by the incoming particle at a wavenumber of $k_{in} = 2.3$, and for comparison, effective potentials corresponding to mean field potentials $gn_0(x)$ and $2gn_0(x)$. Plotted are also exact and WKB approximations to the two stationary scattering solutions, $u(x)$, at this k_{in}: one even and one odd. The reflection coefficient is given by $\sin^2(\delta_e - \delta_o)$ where δ_e (δ_o) is the phase shift of the even (odd) solution. In the WKB-approximation, these phase shifts are identical and full transmission is predicted.